Chiral Perturbation Theory for lattice QCD

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In order to extract predictions of QCD from numerical methods with controlled systematic errors, a lattice formulation is required for which the sources of deviations from QCD are understood and are under control. A significant source of systematic errors for present day lattice simulations are the light quark masses. Even the most powerful computers today do not allow simulations with up- and down-quark masses as light as realized in nature. Instead one simulates with heavier quark masses and fits the analytic predictions obtained in chiral perturbation theory (χPT) to the data. The free parameters in the fit are the low energy couplings of χPT, and once they are determined an extrapolation to small quark masses is possible. Still, to perform the chiral extrapolation the quark masses must be small enough so that χPT is applicable. In practice one would require that next-to-leading order (NLO) χPT describe the data reasonably well.

The present lattice simulations do not meet this requirement. The data do not show the characteristic curvature predicted by NLO χPT. Simulations with even lighter quark masses are required in order to apply χPT with confidence

Lattice simulations with light fermions, especially sea quarks, are computationally demanding and the numerical cost increases substantially with decreasing quark masses. Realistically only the least expensive fermions, Wilson and Kogut-Susskind, can be used on sufficiently large and fine lattices. Lattice fermions with better chiral properties are still too expensive to be used as sea quarks, and this situation is not likely to change in the near future. It is nevertheless expected that the next generation of TFLOP machines will make it possible to generate a few sets of unquenched configurations with sea quarks light enough to be in the chiral regime.

To obtain more information from these configurations they should (and will) be analyzed with various different valence quark masses, i.e. by studying PQ QCD. By including lattice measurements with lighter valence quarks it is possible to penetrate further the chiral regime of QCD. This leads to more data points and would allow more reliable fits of PQ χPT to the lattice data. The reach of such simulations, however, is limited. The cost of light valence quarks also increases with the decreasing mass, and can become prohibitively high for quark masses that are still not very small. This is particularly true for Wilson fermions because of the explicit chiral symmetry breaking by the Wilson term.

An interesting idea for probing the chiral regime is to use different lattice fermions for the valence and sea quarks. In particular, by choosing lattice fermions with good chiral prop-

erties for the valence quarks, the valence quark mass can be made much smaller than in ordinary PQ simulations. A central goal of this strategy is the same as of PQ QCD - to explore a larger portion of the chiral regime by extracting more data points from a given set of unquenched configurations. This should result in more reliable estimates for the low-energy constants of χPT at NLO, the Gasser-Leutwyler coefficients.

In ref. [1] we construct the low-energy chiral effective theory for a "mixed" lattice action correct to $\mathcal{O}(a)$, with explicit dependence on powers of the lattice spacing a by first constructing the appropriate local Symanzik action. There are several reasons for taking this approach. First, the defining non-orthodox feature of the mixed action approach – the use of different Dirac operators for the sea and valence sectors – is purely a lattice artifact. This is a consequence of the fact that by construction all proper lattice fermions reproduce the same continuum physics, and therefore all mixed lattice theories reduce to PQ QCD in the continuum limit. An expansion in a is thus a natural tool to investigate potential peculiarities of the mixed action formulation. Second, a theoretical understanding of the a-dependence in lattice simulations can guide the continuum limit, or allow the extraction of physical information directly from the lattice data, without taking the continuum limit first. Third, χPT provides a useful framework for studying the chiral symmetry breaking due to the discrete space-time lattice.

Currently, we are formulating the χPT for including the lattice effects up to $\mathcal{O}(a^2)$ [2]. There are various reasons for doing this. First of all, the lattice spacings in current unquenched simulations are not very small (≈ 0.2fm), so that neglecting the $\mathcal{O}(a^2)$ contributions might not be justified. Secondly, it is becoming more common to use non-perturbatively improved Wilson fermions in lattice simulations. The leading corrections for these fermions are of $\mathcal{O}(a^2)$ and hence need to be computed in order to know how the continuum limit is approached.

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[1] O. Bär, G. Rupak and N. Shoresh, `hep-lat/0210050`, to be published in Phys. Rev. D.
[2] O. Bär, G. Rupak and N. Shoresh, under publication.